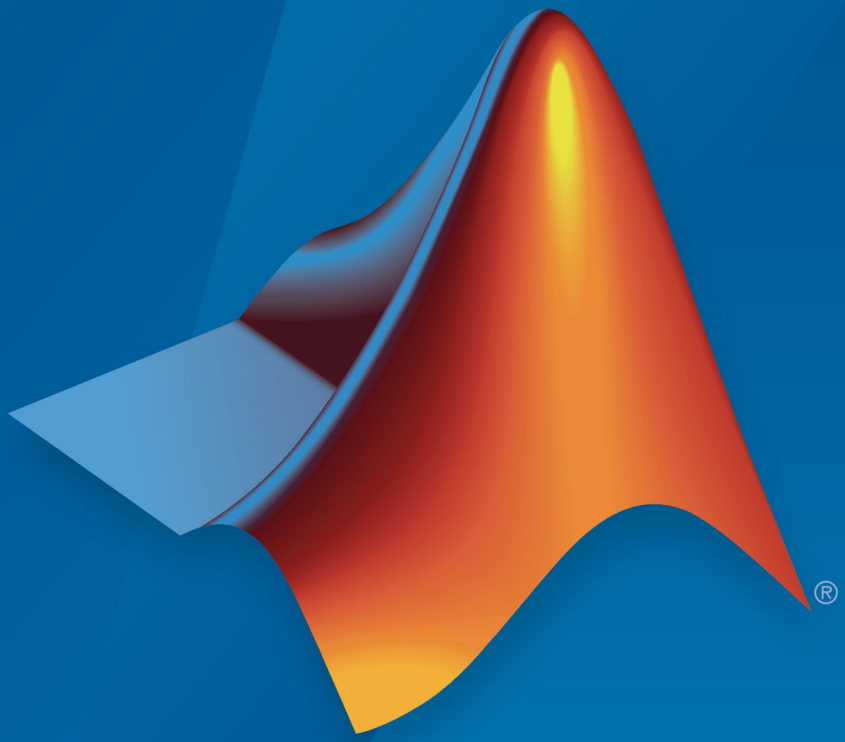
Sensor Fusion and Tracking Toolbox™

User's Guide



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Revision History

September 2018	Online only	New for Version 1.0 (Release 2018b)
March 2019	Online only	Revised for Version 1.1 (Release 2019a)

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1

Tracking Scenarios

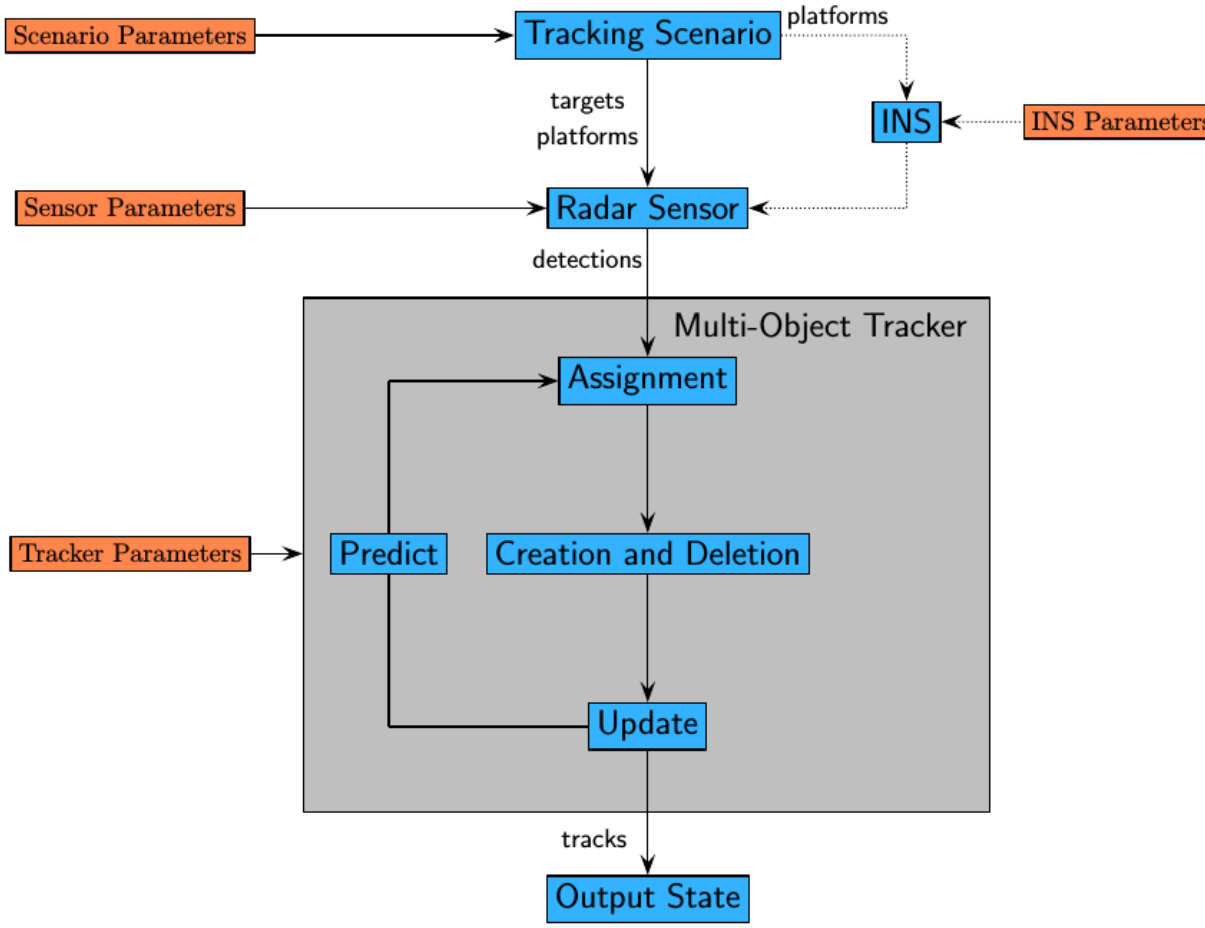
Tracking Simulation Overview

You can build a complete tracking simulation using the functions and objects supplied in this toolbox. The workflow for sensor fusion and tracking simulation consists of three (and optionally four) components. These components are

1. Use the tracking scenario generator to create ground truth for all moving and stationary radar platforms and all target platforms (planes, ships, cars, drones). The `trackingScenario` class models the motion of all platforms in a global coordinate system called scenario coordinates. These objects can represent ships, ground vehicles, airframes, or any object that the radar detects. See “Orientation, Position, and Coordinate Systems” for a discussion of coordinate systems.
2. Optionally, simulate an inertial navigation system (INS) that provides radar sensor platform position, velocity, and orientation relative to scenario coordinates.
3. Create models for each radar sensor with specifications and parameters using the `monostaticRadarSensor`, `radarSensor`, or `radarEmitter` objects. Using target platform pose and profile information, generate synthetic radar detections for each radar-target combination. Methods belonging to `trackingScenario` retrieve the pose and profile of any target platform. The `trackingScenario` generator does not have knowledge of scenario coordinates. It knows the relative positions of the target platforms with respect to the body platform of the radar. Therefore, the detector can only generate detections relative to the radar location and orientation.

   If there is an INS attached to a radar platform, then the radar can transform detections to the scenario coordinate system. The INS allows multiple radars to report detections in a common coordinate system.
4. Process radar detections with a multi-object tracker to associate detections to existing tracks or create tracks. Multi-object tracks include `trackerGNN`, `trackerTOMHT`, `trackerJPDA` and `trackerPHD`. If there is no INS, the tracker can only generate tracks specific to one radar. If an INS is present, the tracker can create tracks using measurements from all radars.

The flow diagram shows the progression of information in a tracking simulation.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Creating a Tracking Scenario

You can define a tracking simulation by using the `trackingScenario` object. By default, the object creates an empty scenario. You can then populate the scenario with platforms by calling the `platform` method as many times as needed. A platform is an object (moving or stationary), which can either be a sensor, a target, or any other entity. A platform can be modeled as a point or a cuboid by specifying the `Dimensions` property of `Platform`. After creating a platform, you can specify the motion of the platform by using its `Trajectory` property. To configure a trajectory, you can use `waypointTrajectory`, which allows you to specify the 3-D waypoints that the platform follows and the associated arrival time for each waypoint. Alternately, you can use `kinematicTrajectory`, which allows you to specify the 3-D acceleration and angular velocity of the platform with initial pose and translational velocity. You can also specify the orientation of a platform using the `Orientation` property of `kinematicTrajectory` or `waypointTrajectory`.

Run the simulation by calling the `advance` method on the `trackingScenario` object in a loop, or by calling the `record` method to run the simulation all at once. You can set the simulation update interval using the `UpdateRate` property in the `trackingScenario` object. You can set the properties of a platform or leave them to their default value. You can set them all except for `PlatformID`. The complete list of `Platform` properties is shown here.

Platform Properties

`PlatformID`	Scenario-defined platform ID.
`ClassID`	User-specified platform classification ID.
`Dimensions`	3-D dimensions of a cuboid that approximates the size of a platform and offset of the origin of the platform body frame from the center of the cuboid. The default value of `Dimensions` has all fields equal to zero, which corresponds to a point model.
`Trajectory`	Platform motion, specified by `kinematicTrajecotry` or `waypointTrajectory`.
`Signatures`	Platform signatures, specified as a cell array of `irSignature`, `rcsSignature`, and `tsSignature` objects. A signature represents the reflection or emission pattern of a platform.
`PoseEstimator`	A pose estimator, specified as a pose-estimator object such as `insSensor` (default).
`Emitter`	Emitters mounted on platform, specified as a cell array of emitter objects, such as `radarEmitter` or `sonarEmitter`.
`Sensors`	Sensors mounted on platform, specified as a cell array of sensor objects such as `irSensor` or `sonarSensor`.

At any time during the simulation, you can retrieve the current values of platform properties using the `platformPoses` and `platformProfiles` methods of the `trackingScenario` object. Both the `platformPoses` and `platformProfiles` methods return properties of all platforms with respect to the scenario's NED frame. You can also use the `pose` method of the `Platform` to return the properties of one specific platform. In addition, the `Platform.targetPoses` method, while similar, returns properties of other platforms with respect to a specified platform.

Create Tracking Scenario with Two Platforms

Construct a tracking scenario with two platforms following different trajectories.

```
sc = trackingScenario('UpdateRate',100.0,'StopTime',1.2);
```

Create two platforms.

```
platfm1 = platform(sc);
platfm2 = platform(sc);
```

Platform 1 follows a circular path of radius 10 m for one second. This is accomplished by placing waypoints in a circular shape, ensuring that the first and last waypoint are the same.

```
wpts1 = [0 10 0; 10 0 0; 0 -10 0; -10 0 0; 0 10 0];
time1 = [0; 0.25; .5; .75; 1.0];
platfm1.Trajectory = waypointTrajectory(wpts1, time1);
```

Platform 2 follows a straight path for one second.

```
wpts2 = [-8 -8 0; 10 10 0];
time2 = [0; 1.0];
platfm2.Trajectory = waypointTrajectory(wpts2,time2);
```

Verify the number of platforms in the scenario.

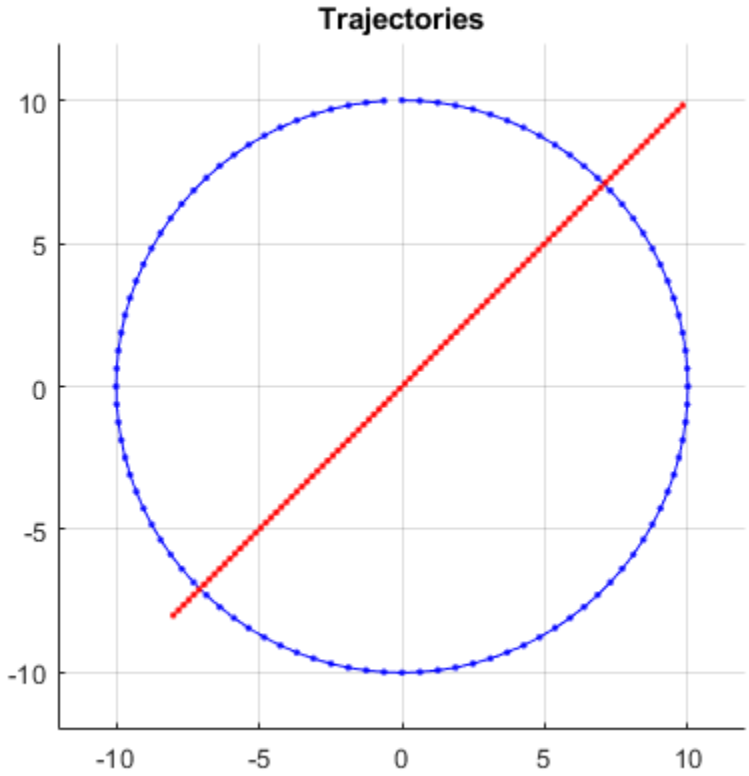
```
disp(sc.Platforms)
```

```
    [1×1 fusion.scenario.Platform]    [1×1 fusion.scenario.Platform]
```

Run the simulation and plot the current position of each platform. Use an animated line to plot the position of each platform

```
figure
grid
axis equal
axis([-12 12 -12 12])
line1 = animatedline('DisplayName','Trajectory 1','Color','b','Marker','.');
line2 = animatedline('DisplayName','Trajectory 2','Color','r','Marker','.');
title('Trajectories')
p1 = pose(platfm1);
p2 = pose(platfm2);
addpoints(line1,p1.Position(1),p1.Position(2));
```

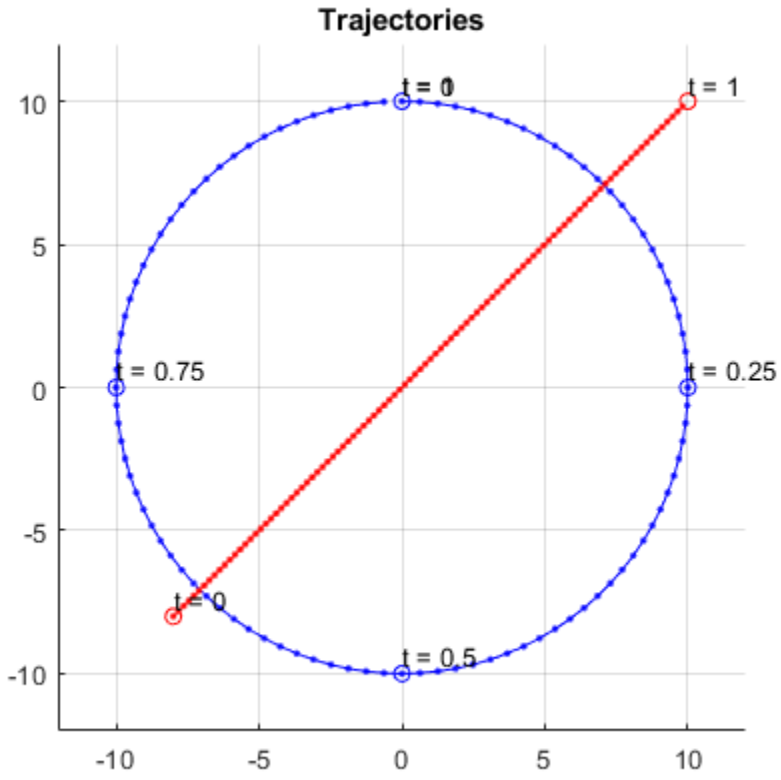
```
addpoints(line2,p2.Position(2),p2.Position(2));

while advance(sc)
    p1 = pose(platfm1);
    p2 = pose(platfm2);
    addpoints(line1,p1.Position(1),p1.Position(2));
    addpoints(line2,p2.Position(2),p2.Position(2));
    pause(0.1)
end
```



Plot the waypoints for both platforms.

```
hold on
plot(wpts1(:,1),wpts1(:,2),' ob')
```

```
text(wpts1(:,1),wpts1(:,2),"t = " + string(time1),'HorizontalAlignment','left','Vertica
plot(wpts2(:,1),wpts2(:,2),' or')
text(wpts2(:,1),wpts2(:,2),"t = " + string(time2),'HorizontalAlignment','left','Vertica
hold off
```



Radar Detections

The radar detectors `monostaticRadarSensor` and `radarSensor` generate measurements from target poses.

Simulate Radar Detections

The `monostaticRadarSensor` object simulates the detection of targets by a scanning radar. You can use the object to model many properties of real radar sensors. For example, you can

- simulate real detections with added random noise
- generate false alarms
- simulate mechanically scanned antennas and electronically scanned phased arrays
- specify angular, range, and range-rate resolution and limits

The radar sensor is assumed to be mounted on a platform and carried by the platform as it maneuvers. A platform can carry multiple sensors. When you create a sensor, you specify sensor positions and orientations with respect to the body coordinate system of a platform. Each call to `monostaticRadarSensor` creates a sensor. The output of `monostaticRadarSensor` generates the detection that can be used as input to multi-object trackers, such as `trackerGNN`, or any tracking filters, such as `trackingKF`.

The radar platform does not maintain any information about the radar sensors that are mounted on it. (The sensor itself contains its position and orientation with respect to the platform on which it is mounted but not which platform). You must create the association between radar sensors and platforms. A way to do this association is to put the platform and its associated sensors into a cell array. When you call a particular sensor, pass in the platform-centric target pose and target profile information. The sensor converts this information to sensor-centric poses. Target poses are outputs of `trackingScenario` methods.

Create Radar Sensor

You can create a radar sensor using the `monostaticRadarSensor` object. Set the radar properties using name-value pairs and then execute the simulator. For example,

```
radar1 = monostaticRadarSensor( ...
    'UpdateRate',updaterate, ...           % Hz
    'ReferenceRange', 111.0e3, ...         % m
    'ReferenceRCS', 0.0, ...               % dBsm
    'HasMechanicalScan',true, ...
    'MaxMechanicalScanRate',scanrate, ...  % deg/s
    'HasElectronicScan',false, ...
    'FieldOfView',fov, ...                 % [az;el] deg
```

```
    'HasElevation',false, ...
    'HasRangeRate',false, ...
    'AzimuthResolution',1.4, ...            % deg
    'RangeResolution', 135.0)                  % m
dets = radar1(targets,simtime);
```

Convenience Syntaxes

There are several syntaxes of `monostaticRadarSensor` that make it easier to specify the properties of commonly implemented radar scan modes. These syntaxes set combinations of these properties: `ScanMode`, `FieldOfView`, `MaxMechanicalScanRate`, `MechanicalScanLimits`, and `ElectronicScanLimits`.

- `sensor = monostaticRadarSensor('Rotator')` creates a `monostaticRadarSensor` object that mechanically scans 360° in azimuth. Setting `HasElevation` to `true` points the radar antenna towards the center of the elevation field of view.
- `sensor = monostaticRadarSensor('Sector')` creates a `monostaticRadarSensor` object that mechanically scans a 90° azimuth sector. Setting `HasElevation` to `true`, points the radar antenna towards the center of the elevation field of view. You can change the `ScanMode` to `'Electronic'` to electronically scan the same azimuth sector. In this case, the antenna is not mechanically tilted in an electronic sector scan. Instead, beams are stacked electronically to process the entire elevation spanned by the scan limits in a single dwell.
- `sensor = monostaticRadarSensor('Raster')` returns a `monostaticRadarSensor` object that mechanically scans a raster pattern spanning 90° in azimuth and 10° in elevation upwards from the horizon. You can change the `ScanMode` property to `'Electronic'` to perform an electronic raster scan in the same volume.
- `sensor = monostaticRadarSensor('No scanning')` returns a `monostaticRadarSensor` object that stares along the radar antenna boresight direction. No mechanical or electronic scanning is performed.

You can set other radar properties when you use these syntaxes. For example,

```
sensor = monostaticRadarSensor('Raster','ScanMode','Electronic')
```

Radar Sensor Parameters

The properties specific to the `monostaticRadarSensor` object are listed here. For more detailed information, type

```
help monostaticRadarSensor
```

at the command line.

Sensor location parameters.

Sensor Location

`SensorIndex`	A unique identifier for each sensor.
`UpdateRate`	Rate at which sensor updates are generated, specified as a positive scalar. The reciprocal of this property must be an integer multiple of the simulation time interval. Updates requested between sensor update intervals do not return detections.
`MountingLocation`	Sensor (x,y,z) defining the offset of the sensor origin from the origin of its platform. The default value positions the sensor origin at the platform origin.
`Yaw`	Angle specifying the rotation around the platform z-axis to align the platform coordinate system with the sensor coordinate system. Positive yaw angles correspond to a clockwise rotation when looking along the positive direction of the z-axis of the platform coordinate system. Rotations are applied using the ZYX convention.
`Pitch`	Angle specifying the rotation around the platform y-axis to align the platform coordinate system with the sensor coordinate system. Positive pitch angles correspond to a clockwise rotation when looking along the positive direction of the y-axis of the platform coordinate system. Rotations are applied using the ZYX convention.

`Roll`	Angle specifying the rotation around the platform x-axis to align the platform coordinate system with the sensor coordinate system. Positive pitch angles correspond to a clockwise rotation when looking along the positive direction of the x-axis of the platform coordinate system. Rotations are applied using the ZYX convention.
`DetectionCoordinates`	Specifies the coordinate system for detections reported in the “Detections” on page 2-20 output `struct`. The coordinate system can be one of: • `'Scenario'` -- detections are reported in the scenario coordinate frame in rectangular coordinates. This option can only be selected when the sensor `HasINS` property is set to `true`. • `'Body'` -- detections are reported in the body frame of the sensor platform in rectangular coordinates. • `'Sensor rectangular'` -- detections are reported in the radar sensor coordinate frame in rectangular coordinates aligned with the sensor frame axes. • `'Sensor spherical'` -- detections are reported in the radar sensor coordinate frame in spherical coordinates based on the sensor frame axes.

Sensitivity parameters.

Sensitivity Parameters

`DetectionProbability`	Probability of detecting a target with radar cross section, `ReferenceRCS`, at the range of `ReferenceRange`.
`FalseAlarmRate`	The probability of a false detection within each resolution cell of the radar. Resolution cells are determined from the `AzimuthResolution` and `RangeResolution` properties and when enabled the `ElevationResolution` and `RangeRateResolution` properties.
`ReferenceRange`	Range at which a target with radar cross section, `ReferenceRCS`, is detected with the probability specified in `DetectionProbability`.
`ReferenceRCS`	The target radar cross section (RCS) in dB at which the target is detected at the range specified by `ReferenceRange` with a detection probability specified by `DetectionProbability`.

Sensor resolution and bias parameters.

Resolution Parameters

`AzimuthResolution`	The radar azimuthal resolution defines the minimum separation in azimuth angle at which the radar can distinguish two targets.
`ElevationResolution`	The radar elevation resolution defines the minimum separation in elevation angle at which the radar can distinguish two targets. This property only applies when the `HasElevation` property is set to `true`.
`RangeResolution`	The radar range resolution defines the minimum separation in range at which the radar can distinguish two targets.
`RangeRateResolution`	The radar range rate resolution defines the minimum separation in range rate at which the radar can distinguish two targets. This property only applies when the `HasRangeRate` property is set to `true`.
`AzimuthBiasFraction`	This property defines the azimuthal bias component of the radar as a fraction of the radar azimuthal resolution specified by the `AzimuthResolution` property. This property sets a lower bound on the azimuthal accuracy of the radar.
`ElevationBiasFraction`	This property defines the elevation bias component of the radar as a fraction of the radar elevation resolution specified by the `ElevationResolution` property. This property sets a lower bound on the elevation accuracy of the radar. This property only applies when the `HasElevation` property is set to `true`.

`RangeBiasFraction`	This property defines the range bias component of the radar as a fraction of the radar range resolution specified by the `RangeResolution` property. This property sets a lower bound on the range accuracy of the radar.
`RangeRateBiasFraction`	This property defines the range rate bias component of the radar as a fraction of the radar range resolution specified by the `RangeRateResolution` property. This property sets a lower bound on the range rate accuracy of the radar. This property only applies when you set the `HasRangeRate` property to `true`.

Enabling parameters.

Enabling Parameters

`HasElevation`	This property allows the radar sensor to scan in elevation and estimate elevation from target detections.
`HasRangeRate`	This property allows the radar sensor to estimate range rate.
`HasFalseAlarms`	This property allows the radar sensor to generate false alarm detection reports.
`HasRangeAmbiguities`	When true, the radar does not resolve range ambiguities. When a radar sensor cannot resolve range ambiguities, targets at ranges beyond the `MaxUnambiguousRange` property value are wrapped into the interval `[0 MaxUnambiguousRange]`. When false, targets are reported at their unwrapped range.
`HasRangeRateAmbiguites`	When true, the radar does not resolve range rate ambiguities. When a radar sensor cannot resolve range rate ambiguities, targets at range rates above the `MaxUnambiguousRadialSpeed` property value are wrapped into the interval `[0 MaxUnambiguousRadialSpeed]`. When false, targets are reported at their unwrapped range rates. This property only applies when the `HasRangeRate` property is set to `true`.

`HasNoise`	Specifies if noise is added to the sensor measurements. Set this property to `true` to report measurements with noise. Set this property to `false` to report measurements without noise. The reported measurement noise covariance matrix contained in the output `objectDetection` struct is always computed regardless of the setting of this property.
`HasOcclusion`	Enable occlusion from extended objects, specified as `true` or `false`. Set this property to `true` to model occlusion from extended objects. Note that both extended objects and point targets can be occluded by extended objects, but a point target cannot occlude another point target or an extended object. Set this property to `false` to disable occlusion of extended objects.
`HasINS`	Set this property to true to enable an optional input argument to pass the current estimate of the sensor platform pose to the sensor. This pose information is added to the `MeasurementParameters` field of the reported detections. Then, the tracking and fusion algorithms can estimate the state of the target detections in scenario coordinates.

Scan parameters.

Scan Parameters

`ScanMode`	This property specifies the scan mode used by the radar as one of: • `'No scanning'` -- the radar does not scan. The radar beam points along the antenna boresight. • `'Mechanical'`-- the radar mechanically scans between the azimuth and elevation limits specified by the `MechanicalScanLimits` property. • `'Electronic'`-- the radar electronically scans between the azimuth and elevation limits specified by the `ElectronicScanLimits` property. • `'Mechanical and electronic'` -- the radar mechanically scans the antenna boresight between the mechanical scan limits and electronically scans beams relative to the antenna boresight between the electronic scan limits. The total field of regard scanned in this mode is the combination of the mechanical and electronic scan limits. In all scan modes except `'No scanning'`, the scan proceeds at angular intervals specified by the radar field of view specified in `FieldOfView`.

`MaxMechanicalScanRate`	This property sets the magnitude of the maximum mechanical scan rate of the radar. When `HasElevation` is `true`, the scan rate is a vector consisting of separate azimuthal and elevation scan rates. When `HasElevation` is `false`, the scan rate is a scalar representing the azimuthal scan rate. The radar sets its scan rate to step the radar mechanical angle by the radar field of regard. When the required scan rate exceeds the maximum scan rate, the maximum scan rate is used.
`MechanicalScanLimits`	This property specifies the mechanical scan limits of the radar with respect to its mounted orientation. When `HasElevation` is `true`, the limits are specified by minimum and maximum azimuth and by minimum and maximum elevation. When `HasElevation` is `false`, limits are specified by minimum and maximum azimuth. Azimuthal scan limits cannot span more than 360 degrees and elevation scan limits must lie in the closed interval `[-90 90]`.
`ElectronicScanLimits`	This property specifies the electronic scan limits of the radar with respect to the current mechanical angle. When `HasElevation` is `true`, the limits are specified by minimum and maximum azimuth and by minimum and maximum elevation. When `HasElevation` is `false`, limits are specified by minimum and maximum azimuth. Both azimuthal and elevation scan limits must lie in the closed interval `[-90 90]`.

`FieldOfView`	This property specifies the sensor azimuthal and elevation fields of view. The field of view defines the total angular extent observed by the sensor during a sensor update. The field of view must lie in the interval *(0,180]*. Targets outside of the sensor angular field of view during a sensor update are not detected.

Range and range rate parameters.

Range and Range Rate Parameters

`MaxUnambiguousRange`	This property specifies the range at which the radar can unambiguously resolve the range of a target. Targets detected at ranges beyond the unambiguous range are wrapped into the range interval `[0 MaxUnambiguousRange]`. This property only applies to true target detections when you set `HasRangeAmbiguities` property to `true`. This property also defines the maximum range at which false alarms are generated. This property only applies to false target detections when you set `HasFalseAlarms` property to `true`.
`MaxUnambiguousRadialSpeed`	This property specifies the maximum magnitude value of the radial speed at which the radar can unambiguously resolve the range rate of a target. Targets detected at range rates whose magnitude is greater than the maximum unambiguous radial speed are wrapped into the range rate interval `[-MaxUnambiguousRadialSpeed MaxUnambiguousRadialSpeed]`. This property only applies to true target detections when you set both the `HasRangeRate` and `HasRangeRateAmbiguities` properties to `true`. This property also defines the range rate interval over which false target detections are generated. This property only applies to false target detections when you set both the `HasFalseAlarms` and `HasRangeRate` properties to `true`.

Detector Input

Each sensor created by `monostaticRadarSensor` accepts as input an array of target structures. This structure serves as the interface between the `trackingScenario` and the sensors. You create the target `struct` from target poses and profile information produced by `trackingScenario` or equivalent software.

The structure contains these fields.

Field	Description
`PlatformID`	Unique identifier for the platform, specified as a scalar positive integer. This is a required field with no default value.
`ClassID`	User-defined integer used to classify the type of target, specified as a nonnegative integer. Zero is reserved for unclassified platform types and is the default value.
`Position`	Position of target in platform coordinates, specified as a real-valued, 1-by-3 vector. This is a required field with no default value. Units are in meters.
`Velocity`	Velocity of target in platform coordinates, specified as a real-valued, 1-by-3 vector. Units are in meters per second. The default is `[0 0 0]`.
`Acceleration`	Acceleration of target in platform coordinates specified as a 1-by-3 row vector. Units are in meters per second-squared. The default is `[0 0 0]`.
`Orientation`	Orientation of the target with respect to platform coordinates, specified as a scalar quaternion or a 3-by-3 rotation matrix. Orientation defines the frame rotation from the platform coordinate system to the current target body coordinate system. Units are dimensionless. The default is `quaternion(1,0,0,0)`.

Field	Description
`AngularVelocity`	Angular velocity of target in platform coordinates, specified as a real-valued, 1-by-3 vector. The magnitude of the vector defines the angular speed. The direction defines the axis of clockwise rotation. Units are in degrees per second. The default is `[0 0 0]`.

You can create a target pose structure by merging information from the platform information output from the `targetProfiles` method of `trackingScenario` and target pose information output from the `targetPoses` method on the platform carrying the sensors. You can merge them by extracting for each `PlatformID` in the target poses array, the profile information in platform profiles array for the same `PlatformID`.

The platform `targetPoses` method returns this structure for each target other than the platform.

Target Poses

`platformID`
`ClassID`
`Position`
`Velocity`
`Yaw`
`Pitch`
`Roll`
`AngularVelocity`

The `platformProfiles` method returns this structure for all platforms in the scenario.

Platform Profiles

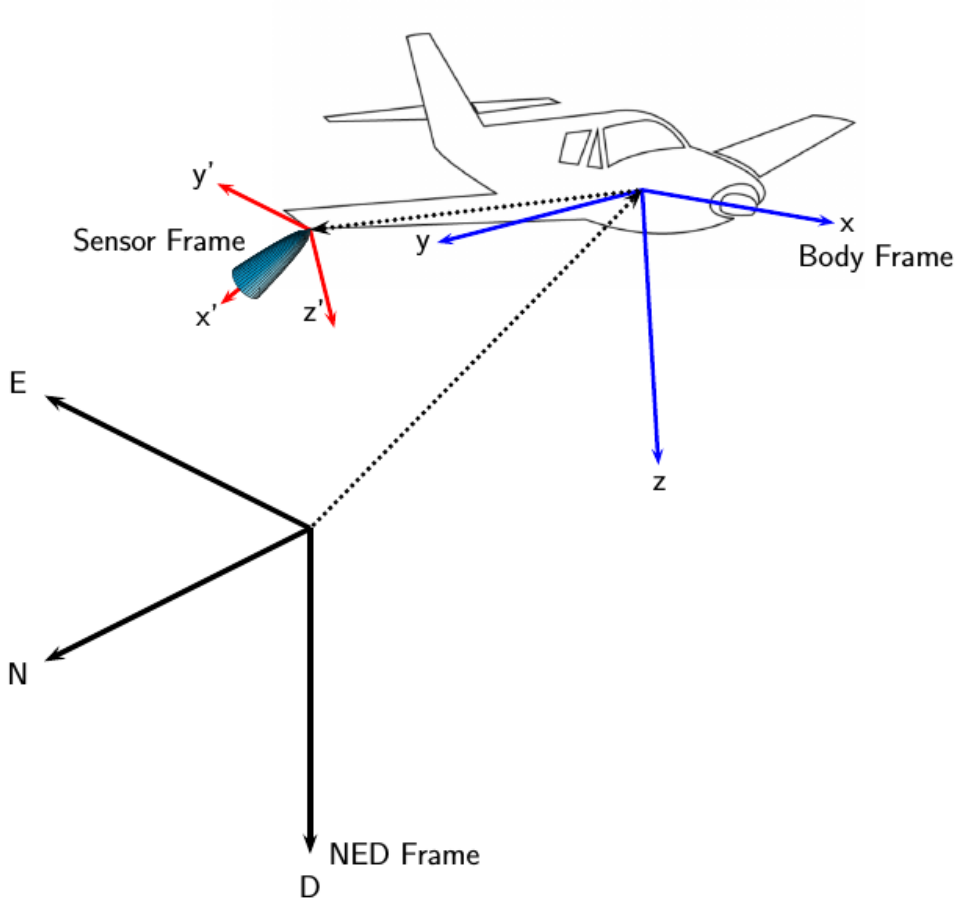
`PlatformID`
`ClassID`
`RCSPattern`
`RCSAzimuthAngles`
`RCSElevationAngles`

Radar Sensor Coordinate Systems

Detections consist of measurements of positions and velocities of targets and their covariance matrices. Detections are constructed with respect to sensor coordinates but can be output in one of several coordinates. Multiple coordinate frames are used to represent the positions and orientations of the various platforms and sensors in a scenario.

In a radar simulation, there is always a top-level global coordinate system which is usually the North-East-Down (NED) Cartesian coordinate system defined by a tangent plane at any point on the surface of the Earth. The `trackingScenario` object models the motion of platforms in the global coordinate system. When you create a platform, you specify its location and orientation relative to the global frame. These quantities define the body axes of the platform. Each radar sensor is mounted on the body of a platform. When you create a sensor, you specify its location and orientation with respect to the platform body coordinates. These quantities define the sensor axes. The body and radar axes can change

over time, however, global axes do not change.



Additional coordinate frames can be required. For example, often tracks are not maintained in NED (or ENU) coordinates, as this coordinate frame changes based on the latitude and longitude where it is defined. For scenarios that cover large areas (over 100 kilometers in each dimension), earth-centered earth-fixed (ECEF) can be a more appropriate global frame to use.

A radar sensor generates measurements in spherical coordinates relative to its sensor frame. However, the locations of the objects in the radar scenario are maintained in a top-level frame. A radar sensor is mounted on a platform and will, by default, only be aware of its position and orientation relative to the platform on which it is mounted. In other words, the radar expects all target objects to be reported relative to the platform body axes. The radar reports the required transformations (position and orientation) to relate the reported detections to the platform body axes. These transformations are used by consumers of the radar detections (e.g. trackers) to maintain tracks in the platform body axes. Maintaining tracks in the platform body axes enables the fusion of measurement or track information across multiple sensors mounted on the same platform.

If the platform is equipped with an inertial navigation system (INS) sensor, then the location and orientation of the platform relative to the top-level frame can be determined. This INS information can be used by the radar to reference all detections to scenario coordinates.

INS

When you specify `HasINS` as true, you must pass in an INS `struct` into the `step` method. This structure consists of the position, velocity, and orientation of the platform in scenario coordinates. These parameters let you express target poses in scenario coordinates by setting the `DetectionCoordinates` property.

Detections

Radar sensor detections are returned as a cell array of `objectDetection` objects. A detection contains these properties.

objectDetection Structure

Field	Definition
`Time`	Measurement time
`Measurement`	Measurements
`MeasurementNoise`	Measurement noise covariance matrix
`SensorIndex`	Unique ID of the sensor
`ObjectClassID`	Object classification
`MeasurementParameters`	Parameters used by initialization functions of any nonlinear Kalman tracking filters
`ObjectAttributes`	Additional information passed to tracker

`Measurement` and `MeasurementNoise` are reported in the coordinate system specified by the `DetectionCoordinates` property of the `monostaticRadarSensor` are reported in sensor Cartesian coordinates.

Measurement Coordinates

<table>
<tr><th>DetectionCoordinates</th><th>Measurement and Measurement Noise Coordinates</th></tr>
<tr><td>'Scenario'
'Body'
'Sensor rectangular'</td><td>Coordinate Dependence on HasRangeRate<table><tr><th>HasRangeRate</th><th>Coordinates</th></tr><tr><td>true</td><td>[x;y;z;vx;vy;vz]</td></tr><tr><td>false</td><td>[x;y;z]</td></tr></table></td></tr>
<tr><td>'Sensor spherical'</td><td>Coordinate Dependence on HasRangeRate and HasElevation<table><tr><th>HasRangeRate</th><th>HasElevation</th><th>Coordinates</th></tr><tr><td>true</td><td>true</td><td>[az;el;rng;rr]</td></tr><tr><td>true</td><td>false</td><td>[az;rng;rr]</td></tr><tr><td>false</td><td>true</td><td>[az;el;rng]</td></tr><tr><td>false</td><td>false</td><td>[az;rng]</td></tr></table></td></tr>
</table>

The `MeasurementParameters` field consists of an array of `struct`s describing a sequence of coordinate transformations from a child frame to a parent frame or the inverse transformations (see “Frame Rotation”). The longest possible sequence of transformations is: Sensor → Platform → Scenario. For example, if the detections are reported in sensor spherical coordinates and `HasINS` is set to false, then the sequence consists of one transformation from sensor to platform. If `HasINS` is true, the sequence of transformations consists of two transformations – first to platform coordinates then to scenario coordinates. Trivially, if the detections are reported in platform rectangular coordinates and `HasINS` is set to false, the transformation consists only of the identity.

Each `struct` takes the form:

MeasurementParameters

Parameter	Definition
`Frame`	Enumerated type indicating the frame used to report measurements. When detections are reported using a rectangular coordinate system, `Frame` is set to `'rectangular'`. When detections are reported in spherical coordinates, `Frame` is set `'spherical'` for the first `struct`.
`OriginPosition`	Position offset of the origin of frame(k) from the origin of frame(k+1) represented as a 3-by-1 vector.
`OriginVelocity`	Velocity offset of the origin of frame(k) from the origin of frame(k+1) represented as a 3-by-1 vector.
`Orientation`	A 3-by-3 real-valued orthonormal frame rotation matrix which rotates the axes of frame(k+1) into alignment with the axes of frame(k).
`IsParentToChild`	A logical scalar indicating if `Orientation` performs a frame rotation from the parent coordinate frame to the child coordinate frame. If false, `Orientation` performs a frame rotation from the child's coordinate frame to the parent's coordinate frame.
`HasElevation`	A logical scalar indicating if the frame has three-dimensional position. Only set to false for the first `struct` when detections are reported in spherical coordinates and `HasElevation` is `false`, otherwise it is `true`.
`HasVelocity`	A logical scalar indicating if the reported detections include velocity measurements. `true` when `HasRangeRate` is enabled, otherwise `false`.

ObjectAttributes

Attribute	Definition
`TargetIndex`	Identifier of the platform, `PlatformID`, that generated the detection. For false alarms, this value is negative.
`SNR`	Detection signal-to-noise ratio in dB.

Multi-Object Tracking

Tracking is the process of estimating the state of motion of an object based on measurements taken off the object. For an object moving in space, the state usually consists of position, velocity, and any other state parameters of objects at any given time. A state is the necessary information needed to predict future states of the system given the specified equations of motion. The estimates are derived from observations on the objects and are updated as new observations are taken. Observations are made using one or more sensors. Observations can only be used to update a track if it is likely that the observation is that of the object having that track. Observations need to be either associated with an existing track or used to create a new track. When several tracks are present, there are several ways observations are associated with one and only one track. The chosen track is based on the "closest" track to the observation.

Tracking and Tracking Filters

Multi-Object Tracking

You can use multi-sensor, multi-target trackers, `trackerGNN`, `trackerJPDA`, and `trackerTOMHT`, to track multiple targets. These trackers implement the multi-object tracking problem using the measurement-to-track association approach. Tracks are initiated and updated using sensor detections of targets. Trackers take several steps when new detections are made:

- The tracker tries to assign a detection to an existing track.
- The tracker creates a track for each detection it cannot assign. When starting the tracker, all detections are used to create tracks.
- The tracker evaluates the status of each track. For new tracks, the status is tentative until enough detections are made to confirm the track. For existing tracks, newly assigned detections are used by the tracking filter to update the track state. When a track has no new added detections, the track is coasted (predicted) until new detections are assigned to it. If no new detections are added after a specified number of updates, the track is deleted.

When tracking multiple objects using these trackers, there are several things to consider:

- Decide which tracker to use.
  - `trackerGNN` uses a global nearest-neighbor assignment algorithm, which maintains a single hypothesis about the tracked object. The tracker offers low computation cost but is not robust during ambiguous association events.
  - `trackerTOMHT` assigns detections based on a track-oriented, multi-hypothesis approach, which maintains multiple hypotheses about the tracked object. The tracker is robust during ambiguous data association events but is computationally more expensive.
  - `trackerJPDA` uses a joint probabilistic data association approach, which applies a soft assignment where multiple detections can contribute to each track. The tracker balances the robustness and computation cost between `trackerGNN` and `trackerTOMHT`.

  See the “Tracking Closely Spaced Targets Under Ambiguity” example for a comparison between these three trackers.

- Decide which type of tracking filter to use.

  The choice of tracking filter depends on the expected dynamics of the object you want to track. The toolbox provides multiple Kalman filters including the Linear Kalman filter, `trackingKF`, the Extended Kalman filter, `trackingEKF`, the Unscented Kalman filter, `trackingUKF`, and the Cubature Kalman filter, `trackingCKF`. The linear Kalman filter is used when the dynamics of the object follow a linear model and the measurements are linear functions of the state vector. The extended, unscented, and cubature Kalman filters are used when the dynamics are nonlinear, the measurement model is nonlinear, or both. The toolbox also provides non-Gaussian filters such as the particle filter, `trackingPF`, Gaussian-sum filter, `trackingGSF`, and the Interacting Multiple Model (IMM) filter, `trackingIMM`. See the "Tracking with Range-Only Measurements" and "Tracking Maneuvering Targets" examples for more information about these filters.

  You can set the type of filter by specifying the `FilterInitializationFcn` property of a tracker. For example, if you set the `FilterInitializationFcn` property to `@initcaekf`, then the tracker uses the `initcaekf` function to create a constant-acceleration extended Kalman filter for a new track generated from detections.

- Decide which track logic to use.

  You can specify the conditions under which a track is confirmed or deleted by setting the `TrackLogic` property. Three algorithms are supported:

  - `'History'` — Track confirmation and deletion are based on the number of times the track has been assigned to a detection in the last several tracker updates. You can use this logic with `trackerGNN` and `trackerJPDA`.
  - `'Score'` — Track confirmation and deletion are based on a log-likelihood computation. A high score means that the track is more likely to be valid. A low score means that the track is more likely to be false. You can use this logic with `trackerGNN` and `trackerTOMHT`.
  - `'Integrated'` — Track confirmation and deletion are based on the probability of track existence. You can use this logic with `trackerJPDA`.

  For more details, see the "Introduction to Track Logic" example.

You can also use a multi-sensor, multi-target tracker, `trackerPHD`, to track multiple targets simultaneously. `trackerPHD` approaches the multi-object tracking problem using the random finite set (RFS) method and tracks the probability hypothesis density (PHD) of a scenario. `trackerPHD` extracts peaks from the PHD-intensity to represent potential

targets and maintain identities of targets by assigning a label to each component. The toolbox offers one realization of PHD, `ggiwphd`, which represents the PHD of extended targets using a Gamma Gaussian Inverse-Wishart (GGIW) target-state model. You can represent the configurations of sensors for `trackerPHD` using `trackingSensorConfiguration`.

Multi-Object Tracker Properties

trackerGNN Properties

The `trackerGNN` object is a multi-sensor, multi-object tracker that uses global nearest neighbor association. Each detection can be assigned to only one track (single-hypothesis tracker) which can also be a new track that the detection initiates. At each step of the simulation, the tracker updates the track state. You can specify the behavior of the tracker by setting the following properties.

trackerGNN Properties

`FilterInitializationFcn`	A handle to a function that initializes a tracking filter based on a single detection. This function is called when a detection cannot be assigned to an existing track. For example, `initcaekf` creates an extended Kalman filter for an accelerating target. All tracks are initialized with the same type of filter.
`Assignment`	The name of the assignment algorithm. The tracker provides three built-in algorithms: `'Munkres'`, `'Jonker-Volgenant'`, and `'Auction'` algorithms. You can also create your own custom assignment algorithm by specifying `'Custom'`.
`CustomAssignmentFcn`	The name of the custom assignment algorithm function. This property is available on when the `Assignment` property is set to `'Custom'`.
`AssignmentThreshold`	Specify the threshold that controls the assignment of a detection to a track. Detections can only be assigned to a track if their normalized distance from the track is less than the assignment threshold. Each tracking filter has a different method of computing the normalized distance. Increase the threshold if there are detections that can be assigned to tracks but are not. Decrease the threshold if there are detections that are erroneously assigned to tracks.

`TrackLogic`	Specify the track confirmation logic --`'History'` or `'Score'`. For descriptions of these options, type
	
	`help trackHistoryLogic`
	
	or
	
	`help trackScoreLogic`
	
	at the command line.

`ConfirmationThreshold`	Specify the threshold for track confirmation. The threshold depends on the setting for `TrackLogic` • `'History'` -- specify the confirmation threshold as `[M N]`. If the track is detected at least `M` times in the last `N` updates, the track is confirmed. • `'Score'` --- specify the confirmation threshold as a single number. If the score is greater than or equal to the threshold, this track is confirmed. .

`DeletionThreshold`	Specify the threshold for track deletion. The threshold depends on the setting of `TrackLogic` • `'History'` -- specify the deletion threshold as a pair of integers `[P R]`. A track is deleted if it is not assigned to a track at least `P` times in the last `R` updates. • `'Score'` --- specify the deletion threshold as a single number. The track is deleted if its score decreases by at least this threshold from its maximum track score. .
`DetectionProbability`	Specify the probability of detection as a number in the range *(0,1)*. The probability of detection is used to calculate the track score when initializing and updating a track. This property is used only when `TrackLogic` is set to `'Score'`.
`FalseAlarmRate`	Specify the rate of false detection as a number in the range *(0,1)*. The false alarm rate is used to calculate the track score when initializing and updating a track. This property is used only when `TrackLogic` is set to `'Score'`.
`Beta`	Specify the rate of new tracks per unit volume as a positive number. This property is used only when `TrackLogic` is set to `'Score'`. The rate of new tracks is used in calculating the track score during track initialization. This property is used only when `TrackLogic` is set to `'Score'`.

`Volume`	Specify the volume of the sensor measurement bin as a positive scalar. For example, a radar sensor that produces a 4-D measurement of azimuth, elevation, range, and range-rate creates a 4-D volume. The volume is a product of the radar angular beamwidth, the range bin width, and the range-rate bin width. The volume is used in calculating the track score when initializing and updating a track. This property is used only when `TrackLogic` is set to `'Score'`.
`MaxNumTracks`	Specify the maximum number of tracks the tracker can maintain.
`MaxNumSensors`	Specify the maximum number of sensors sending detections to the tracker as a positive integer. This number must be greater than or equal to the largest `SensorIndex` value used in the `objectDetection` input to the `step` method. This property determines how many sets of `ObjectAttributes` each track can have.
`HasDetectableTrackIDsInput`	Set this property to `true` if you want to provide a list of detectable track IDs as input to the `step` method. This list contains all tracks that the sensors expect to detect and, optionally, the probability of detection for each track ID.
`HasCostMatrixInput`	Set this property to `true` if you want to provide an assignment cost matrix as input to the `step` method.

trackerGNN Input

The input to the `trackerGNN` consists of a list of detections, the update time, cost matrix, and other data. Detections are specified as a cell array of `objectDetection` objects (see “Detections” on page 2-20). The input arguments are listed here.

trackerGNN Input

`tracker`	A `trackerGNN` object.
`detections`	Cell array of `objectDetection` objects (see “Detections” on page 2-20).
`time`	Time to which all the tracks are to be updated and predicted. The time at this execution step must be greater than the value in the previous call.
`costmatrix`	Cost matrix for assigning detections to tracks. A real *T*-by-*D* matrix, where *T* is the number of tracks listed in the `allTracks` argument returned from the previous call to `step`. *D* is the number of detections that are input in the current call. A larger cost matrix entry means a lower likelihood of assignment.
`detectableTrackIDs`	IDs of tracks that the sensors expect to detect, specified as an *M*-by-1 or *M*-by-2 matrix. The first column consists of track IDs, as reported in the `TrackID` field of the tracker output. The second column is optional and allows you to add the detection probability for each track.

trackerGNN Output

The output of the tracker can consist of up to three `struct` arrays with track state information. You can retrieve just the confirmed tracks, the confirmed and tentative tracks, or these tracks plus a combined list of all tracks.

```
confirmedTracks = step(...)

[confirmedTracks, tentativeTracks] = step(...)

[confirmedTracks, tentativeTracks, allTracks] = step(...)
```

The fields contained in the `struct` are:

trackerGNN Output `struct`

`TrackID`	Unique integer that identifies the track.
`UpdateTime`	Time to which the track is updated.
`Age`	Number of updates since track initialization.
`State`	State vector at update time.
`StateCovariance`	State covariance matrix at update time.
`IsConfirmed`	True if the track is confirmed.
`TrackLogic`	The track logic used in confirming the track – `'History'` or `'Score'`.
`TrackLogicState`	The current state of the track logic. • For `'History'` track logic, a 1-by-*Q* logical array, where *Q* is the larger of *N* specified in the confirmation threshold property, `ConfirmationThreshold`, and *R* specified in the deletion threshold property, `DeletionThreshold`. • For `'Score'` track logic, a 1-by-2 numerical array in the form: `[currentScore, maxScore]`.
`IsCoasted`	True if the track has been updated without a detection. In this case, tracks are predicted to the current time.
`ObjectClassID`	An integer value representing the target classification. Zero is reserved for an "unknown" class.
`ObjectAttributes`	A cell array of cells. Each cell captures the object attributes reported by the corresponding sensor.

Multiple Extended Object Tracking

In traditional tracking systems, the point target model is commonly used. In a point target model:

- Each object is modeled as a point without any spatial extent.
- Each object gives rise to at most one measurement per sensor scan.

Though the point target model simplifies tracking systems, the assumptions above may not be valid when modern tracking systems are considered:

- In modern tracking systems, the dimensions of the extended object play a significant role. For example, in autonomous vehicles, target dimensions must be considered properly to avoid collision with objects around the autonomous system.
- Modern sensors have a high resolution, and an object can occupy more than one resolution cell. As a result, the sensor may report multiple detections for that object. In this case, In this case, the point model cannot fully exploit the sensor ability to detect object extent.

In extended object tracking, a sensor can return multiple detections per scan for an extended object. The differences between extended object tracking and point object tracking are more about the sensor properties rather than object properties. For example, if the resolution of a sensor is high enough, even an object with small dimensions can still occupy several resolution cells of the sensor.

Sensor Fusion and Tracking Toolbox offers several methods and examples for multiple extended object tracking. Depending on the assumptions made in the detection and tracker, these methods can be separated into the following categories:

- One detection per object.

  In this category, the conventional trackers (such as `trackerGNN`, `trackerJPDA`, and `trackerTOMHT`) are used, which assume one detection per object. This category can further be divided into two methods:

  - A point detection per object.

    In this method, even though the sensor returns multiple detections per object, these detections are first converted into one representative point detection with certain covariance to account for the distribution of these detections. Then the representative point detection is processed by a conventional tracker, which

models the object as a point target and tracks its kinematic state. Even though this method is simple to use, it overlooks the ability of the sensor to detect the object dimension.

The Point Object Tracker approach shown in the first part of “Extended Object Tracking” example adopts this method.

- An extended object detection per object.

  In this method, the multiple detections of an extended object are converted into a single parameterized shape detection. The shape detection includes the kinematic states of the object, as well as its extent parameters such as length, width and height. Then the shape detection is processed by a conventional tracker, which models the object as an extended object by tracking both the object kinematic state and its dimensions.

  In the “Track Vehicles Using Lidar: From Point Cloud to Track List” example, the Lidar detections of each vehicle are converted into a cuboid detection with length, width, and height. A JPDA tracker is used to track the position, velocity and dimensions for all the vehicles with these cuboid detections.

- Multiple detections per object.

  In this category, extended object trackers (such as `trackerPHD`) are used, which assume multiple detections per object. The detections are fed directly to the tracker, and the tracker models the extended object using certain default geometric shapes with variable sizes.

  In the “Extended Object Tracking” example, the GGIW-PHD Extended Object Tracker approach represents vehicle shapes as ellipses, and the Prototype Extended Object Tracker approach represents vehicle shapes as rectangles.

  In the “Marine Surveillance Using a PHD Tracker” example, the GGIW-PHD tracker models the ship shapes as ellipses.

Linear Kalman Filters

In this section...

When you use a Kalman filter to track objects, you use a sequence of detections or measurements to construct a model of the object motion. Object motion is defined by the evolution of the state of the object. The Kalman filter is an optimal, recursive algorithm for estimating the track of an object. The filter is recursive because it updates the current state using the previous state, using measurements that may have been made in the interval. A Kalman filter incorporates these new measurements to keep the state estimate as accurate as possible. The filter is optimal because it minimizes the mean-square error of the state. You can use the filter to predict future states or estimate the current state or past state.

State Equations

For most types of objects tracked in Sensor Fusion and Tracking Toolbox, the state vector consists of one-, two- or three-dimensional positions and velocities.

Start with Newton equations for an object moving in the x-direction at constant acceleration and convert these equations to space-state form.

$$m\ddot{x} = f$$

$$\ddot{x} = \frac{f}{m} = a$$

If you define the state as

$$x_1 = x$$

$$x_2 = \dot{x},$$

you can write Newton’s law in state-space form.

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}a$$

You use a linear dynamic model when you have confidence that the object follows this type of motion. Sometimes the model includes process noise to reflect uncertainty in the motion model. In this case, Newton's equations have an additional term.

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}a + \begin{bmatrix}0\\1\end{bmatrix}v_k$$

v_k and is the unknown noise perturbations of the acceleration. Only the statistics of the noise are known. It is assumed to be zero-mean Gaussian white noise.

You can extend this type of equation to more than one dimension. In two dimensions, the equation has the form

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\y_1\\y_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\y_1\\y_2\end{bmatrix} + \begin{bmatrix}0\\a_x\\0\\a_y\end{bmatrix} + \begin{bmatrix}0\\v_x\\0\\v_y\end{bmatrix}$$

The 4-by-4 matrix on the right side is the state transition model matrix. For independent *x*- and *y*- motions, this matrix is block diagonal.

When you transition to discrete time, you integrate the equations of motion over the length of the time interval. In discrete form, for a sample interval of T, the state-representation becomes

$$\begin{bmatrix}x_{1,k+1}\\x_{2,k+1}\end{bmatrix} = \begin{bmatrix}1 & T\\0 & 1\end{bmatrix}\begin{bmatrix}x_{1,k}\\x_{2,k}\end{bmatrix} + \begin{bmatrix}0\\T\end{bmatrix}a + \begin{bmatrix}0\\1\end{bmatrix}\tilde{v}$$

The quantity x_{k+1} is the state at discrete time $k+1$, and x_k is the state at the earlier discrete time, k. If you include noise, the equation becomes more complicated, because the integration of noise is not straightforward.

The state equation can be generalized to

$$x_{k+1} = F_k x_k + G_k u_k + v_k$$

F_k is the state transition matrix and G_k is the control matrix. The control matrix takes into account any known forces acting on the object. Both of these matrices are given. The last

term represents noise-like random perturbations of the dynamic model. The noise is assumed to be zero-mean Gaussian white noise.

Continuous-time systems with input noise are described by linear stochastic differential equations. Discrete-time systems with input noise are described by linear stochastic differential equations. A state-space representation is a mathematical model of a physical system where the inputs, outputs, and state variables are related by first-order coupled equations.

Measurement Models

Measurements are what you observe about your system. Measurements depend on the state vector but are not always the same as the state vector. For instance, in a radar system, the measurements can be spherical coordinates such as range, azimuth, and elevation, while the state vector is the Cartesian position and velocity. For the linear Kalman filter, the measurements are always linear functions of the state vector, ruling out spherical coordinates. To use spherical coordinates, use the extended Kalman filter.

The measurement model assumes that the actual measurement at any time is related to the current state by

$$z_k = H_k x_k + w_k$$

w_k represents measurement noise at the current time step. The measurement noise is also zero-mean white Gaussian noise with covariance matrix Q described by $Q_k = E[n_k n_k^T]$.

Linear Kalman Filter Equations

Without noise, the dynamic equations are

$$x_{k+1} = F_k x_k + G_k u_k .$$

Likewise, the measurement model has no measurement noise contribution. At each instance, the process and measurement noises are not known. Only the noise statistics are known. The

$$z_k = H_k x_k$$

You can put these equations into a recursive loop to estimate how the state evolves and also how the uncertainties in the state components evolve.

Filter Loop

Start with a best estimate of the state, $x_{0/0}$, and the state covariance, $P_{0/0}$. The filter performs these steps in a continual loop.

1. Propagate the state to the next step using the motion equations.

   $$x_{k+1|k} = F_k x_{k|k} + G_k u_k .$$

   Propagate the covariance matrix as well.

   $$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k .$$

   The subscript notation $k+1|k$ indicates that the quantity is the optimum estimate at the $k+1$ step propagated from step k. This estimate is often called the *a priori* estimate.

   Then predict the measurement at the updated time.

   $$z_{k+1|k} = H_{k+1} x_{k+1|k}$$

2. Use the difference between the actual measurement and predicted measurement to correct the state at the updated time. The correction requires computing the Kalman gain. To do this, first compute the measurement prediction covariance (innovation)

   $$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1}$$

   Then the Kalman gain is

   $$K_{k+1} = P_{k+1|k} H_{k+1}^T S_{k+1}^{-1}$$

   and is derived from using an optimality condition.

3. Correct the predicted estimate with the measurement. Assume that the estimate is a linear combination of the predicted state and the measurement. The estimate after correction uses the subscript notation, $k+1|k+1$. is computed from

   $$x_{k+1|k+1} = x_{k+1|k} + K_{k+1}(z_{k+1} - z_{k+1|k})$$

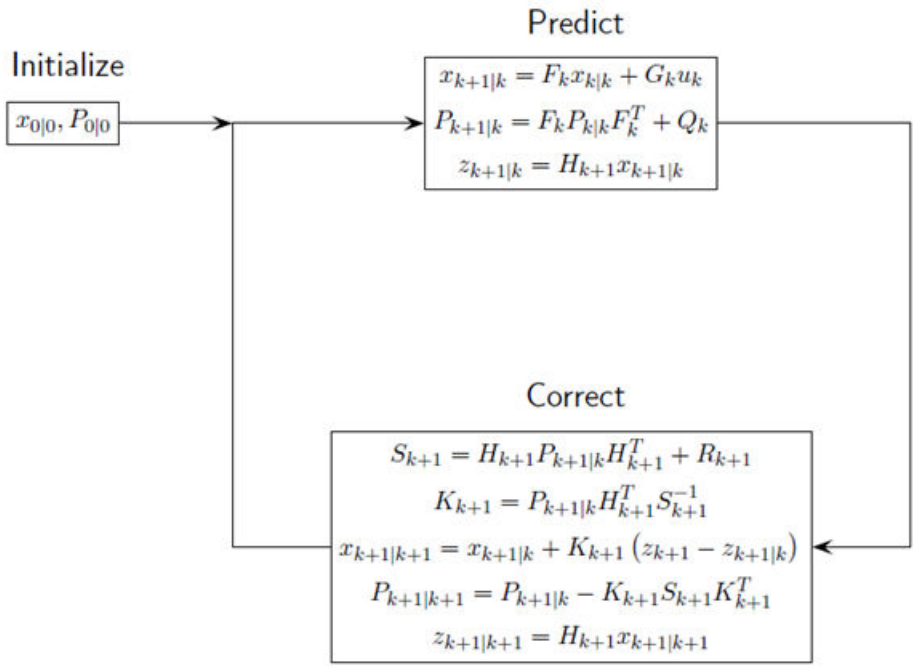
   where K_{k+1} is the Kalman gain. The corrected state is often called the *a posteriori* estimate of the state because it is derived after the measurement is included.

   Correct the state covariance matrix

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}S_{k+1}K'_{k+1}$$

Finally, you can compute a measurement based upon the corrected state. This is not a correction to the measurement but is a best estimate of what the measurement would be based upon the best estimate of the state. Comparing this to the actual measurement gives you an indication of the performance of the filter.

This figure summarizes the Kalman loop operations.



Constant Velocity Model

The linear Kalman filter contains a built-in linear constant-velocity motion model. Alternatively, you can specify the transition matrix for linear motion. The state update at the next time step is a linear function of the state at the present time. In this filter, the

measurements are also linear functions of the state described by a measurement matrix. For an object moving in 3-D space, the state is described by position and velocity in the *x*-, *y*-, and *z*-coordinates. The state transition model for the constant-velocity motion is

$$\begin{bmatrix} x_{k+1} \\ v_{x,k+1} \\ y_{k+1} \\ v_{y,k+1} \\ z_{k+1} \\ v_{z,k+1} \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v_{x,k} \\ y_k \\ v_{y,k} \\ z_k \\ v_{z,k} \end{bmatrix}$$

The measurement model is a linear function of the state vector. The simplest case is one where the measurements are the position components of the state.

$$\begin{bmatrix} m_{x,k} \\ m_{y,k} \\ m_{z,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ v_{x,k} \\ y_k \\ v_{y,k} \\ z_k \\ v_{z,k} \end{bmatrix}$$

Constant Acceleration Model

The linear Kalman filter contains a built-in linear constant-acceleration motion model. Alternatively, you can specify the transition matrix for constant-acceleration linear motion. The transition model for linear acceleration is

$$\begin{bmatrix} x_{k+1} \\ v_{x,k+1} \\ a_{x,k+1} \\ y_{k+1} \\ v_{y,k+1} \\ a_{y,k+1} \\ z_{k+1} \\ v_{z,k+1} \\ a_{z,k+1} \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & \frac{1}{2}T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & \frac{1}{2}T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v_{x,k} \\ a_{x,k} \\ y_k \\ v_{y,k} \\ a_{y,k} \\ z_k \\ v_{z,k} \\ a_{z,k} \end{bmatrix}$$

The simplest case is one where the measurements are the position components of the state.

$$\begin{bmatrix} m_{x,k} \\ m_{y,k} \\ m_{z,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ v_{x,k} \\ a_{x,k} \\ y_k \\ v_{y,k} \\ a_{y,k} \\ z_k \\ v_{z,k} \\ a_{y,k} \end{bmatrix}$$

Extended Kalman Filters

In this section...

Use an extended Kalman filter when object motion follows a nonlinear state equation or when the measurements are nonlinear functions of the state. A simple example is when the state or measurements of the object are calculated in spherical coordinates, such as azimuth, elevation, and range.

State Update Model

The extended Kalman filter formulation linearizes the state equations. The updated state and covariance matrix remain linear functions of the previous state and covariance matrix. However, the state transition matrix in the linear Kalman filter is replaced by the Jacobian of the state equations. The Jacobian matrix is not constant but can depend on the state itself and time. To use the extended Kalman filter, you must specify both a state transition function and the Jacobian of the state transition function.

Assume there is a closed-form expression for the predicted state as a function of the previous state, controls, noise, and time.

$$x_{k+1} = f(x_k, u_k, w_k, t)$$

The Jacobian of the predicted state with respect to the previous state is

$$F^{(x)} = \frac{\partial f}{\partial x}.$$

The Jacobian of the predicted state with respect to the noise is

$$F^{(w)} = \frac{\partial f}{\partial w_i}.$$

These functions take simpler forms when the noise enters linearly into the state update equation:

$$x_{k+1} = f(x_k, u_k, t) + w_k$$

In this case, $F^{(w)} = 1_M$.

Measurement Model

In the extended Kalman filter, the measurement can be a nonlinear function of the state and the measurement noise.

$$z_k = h(x_k, v_k, t)$$

The Jacobian of the measurement with respect to the state is

$$H^{(x)} = \frac{\partial h}{\partial x}.$$

The Jacobian of the measurement with respect to the measurement noise is

$$H^{(v)} = \frac{\partial h}{\partial v}.$$

These functions take simpler forms when the noise enters linearly into the measurement equation:

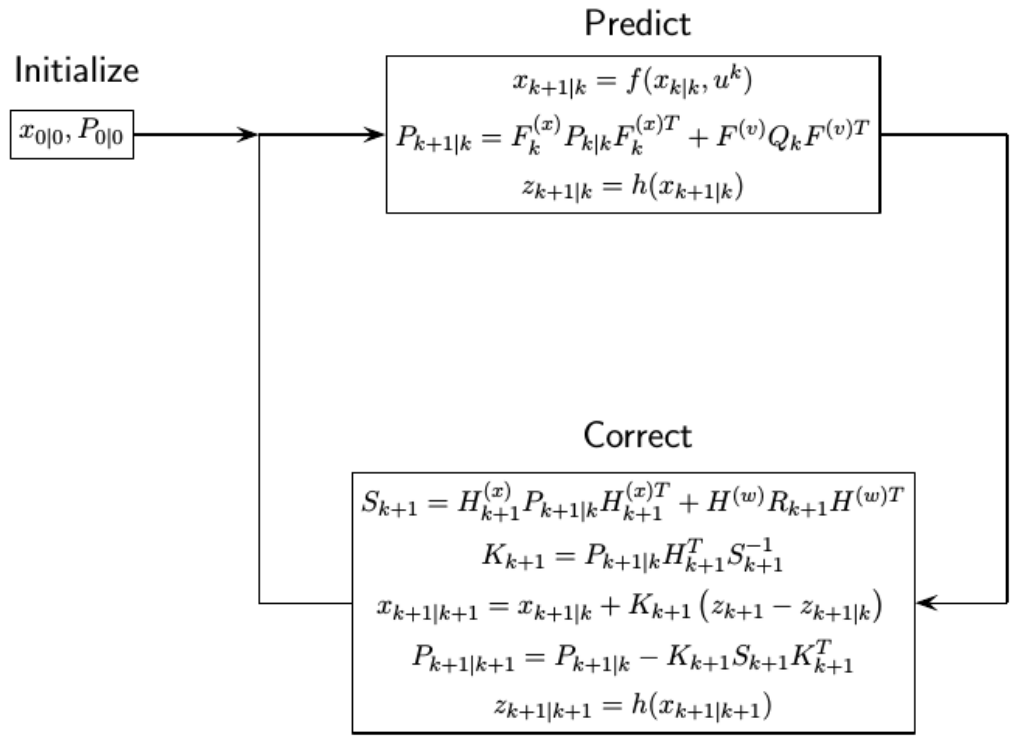
$$z_k = h(x_k, t) + v_k$$

In this case, $H^{(v)} = 1_N$.

Extended Kalman Filter Loop

This is extended kalman filter loop is almost identical to the linear Kalman filter loop except that:

- The exact nonlinear state update and measurement functions are used whenever possible and the state transition matrix is replaced by the state Jacobian
- The measurement matrices are replaced by the appropriate Jacobians.



Predefined Extended Kalman Filter Functions

Sensor Fusion and Tracking Toolbox provides predefined state update and measurement functions to use in the extended Kalman filter.

Motion Model	Function Name	Function Purpose
Constant velocity	`constvel`	Constant-velocity state update model
	`constveljac`	Constant-velocity state update Jacobian

Motion Model	Function Name	Function Purpose
	`cvmeas`	Constant-velocity measurement model
	`cvmeasjac`	Constant-velocity measurement Jacobian
Constant acceleration	`constacc`	Constant-acceleration state update model
	`constaccjac`	Constant-acceleration state update Jacobian
	`cameas`	Constant-acceleration measurement model
	`cameasjac`	Constant-acceleration measurement Jacobian
Constant turn rate	`constturn`	Constant turn-rate state update model
	`constturnjac`	Constant turn-rate state update Jacobian
	`ctmeas`	Constant turn-rate measurement model
	`ctmeasjac`	Constant-turnrate measurement Jacobian

4

Data Structures

Target Pose

Target pose consists of the position, velocity, orientation, and signature of a target. All quantities are specified in the frame of a sensor platform. The target pose structure has these fields:

Field	Description
`PlatformID`	Unique identifier for the platform, specified as a scalar positive integer. This is a required field with no default value.
`ClassID`	User-defined integer used to classify the type of target, specified as a nonnegative integer. Zero is reserved for unclassified platform types and is the default value.
`Position`	Position of target in platform coordinates, specified as a real-valued, 1-by-3 vector. This is a required field with no default value. Units are in meters.
`Velocity`	Velocity of target in platform coordinates, specified as a real-valued, 1-by-3 vector. Units are in meters per second. The default is `[0 0 0]`.
`Acceleration`	Acceleration of target in platform coordinates specified as a 1-by-3 row vector. Units are in meters per second-squared. The default is `[0 0 0]`.
`Orientation`	Orientation of the target with respect to platform coordinates, specified as a scalar quaternion or a 3-by-3 rotation matrix. Orientation defines the frame rotation from the platform coordinate system to the current target body coordinate system. Units are dimensionless. The default is `quaternion(1,0,0,0)`.

Field	Description
`AngularVelocity`	Angular velocity of target in platform coordinates, specified as a real-valued, 1-by-3 vector. The magnitude of the vector defines the angular speed. The direction defines the axis of clockwise rotation. Units are in degrees per second. The default is `[0 0 0]`.

Platform Pose

Platform pose consists of the position, velocity, orientation, and angular velocity of a platform with respect to scenario coordinates. The returned structure has these fields:

Field	Description
`PlatformID`	Unique identifier for the platform, specified as a scalar positive integer. This is a required field with no default value.
`ClassID`	User-defined integer used to classify the type of target, specified as a nonnegative integer. Zero is reserved for unclassified platform types and is the default value.
`Position`	Position of target in scenario coordinates, specified as a real-valued 1-by-3 vector. This is a required field with no default value. Units are in meters.
`Velocity`	Velocity of platform in scenario coordinates, specified as a real-valued 1-by-3 vector. Units are in meters per second. The default value is `[0 0 0]`.
`Acceleration`	Acceleration of the platform in scenario coordinates, specified as a 1-by-3 row vector in meters per second-squared. The default value is `[0 0 0]`.
`Orientation`	Orientation of the platform with respect to the local scenario NED coordinate frame, specified as a scalar quaternion or a 3-by-3 rotation matrix. The format is specified by the `fmt` input argument. Orientation defines the frame rotation from the local NED coordinate system to the current platform body coordinate system. Units are dimensionless. The default value is `quaternion(1,0,0,0)`.

Field	Description
`AngularVelocity`	Angular velocity of the platform in scenario coordinates, specified as a real-valued 1-by-3 vector. The magnitude of the vector defines the angular speed. The direction defines the axis of clockwise rotation. Units are in degrees per second. The default is value `[0 0 0]`.

Platform Profiles

A profile contains the radar, IR, or sonar properties of a platform. The structure contains these fields:

Field	Description
`PlatformID`	Scenario-defined platform identifier, defined as a positive integer
`ClassID`	User-defined platform classification identifier, defined as a nonnegative integer
`Signatures`	Platform signatures defined as a cell array of radar cross-section (`rcsSignature`), IR emission pattern (`irSignature`), and sonar target strength (`tsSignature`) objects.

Object Detections

Sensor detections are returned as a cell array of `objectDetection` objects. A detection contains these properties.

Property	Definition
`Time`	Measurement time
`Measurement`	Object measurements
`MeasurementNoise`	Measurement noise covariance matrix
`SensorIndex`	Unique ID of the sensor
`ObjectClassID`	Object classification
`MeasurementParameters`	Parameters used by initialization functions of nonlinear Kalman tracking filters
`ObjectAttributes`	Additional information passed to tracker

Measurements and Measurement Noise

The sensor measures the coordinates of the target. The `Measurement` and `MeasurementNoise` values are reported in the coordinate system specified by the `DetectionCoordinates` property of the sensor.

When the `DetectionCoordinates` property is `'Scenario'`, `'Body'`, or `'Sensor rectangular'`, the `Measurement` and `MeasurementNoise` values are reported in rectangular coordinates. Velocities are only reported when the range rate property, `HasRangeRate`, is `true`.

When the `DetectionCoordinates` property is `'Sensor spherical'`, the `Measurement` and `MeasurementNoise` values are reported in a spherical coordinate system derived from the sensor rectangular coordinate system. Elevation and range rate are only reported when `HasElevation` and `HasRangeRate` are `true`.

Measurements are ordered as [azimuth, elevation, range, range rate]. Reporting of elevation and range rate depends on the corresponding `HasElevation` and `HasRangeRate` property values. Angles are in degrees, range is in meters, and range rate is in meters per second.

Measurement Coordinates

<table>
<tr><th>DetectionCoordinates</th><th>Measurement and Measurement Noise Coordinates</th></tr>
<tr><td>'Scenario'
'Body'
'Sensor rectangular'</td><td>Coordinate Dependence on HasRangeRate<table><tr><th>HasRangeRate</th><th>Coordinates</th></tr><tr><td>true</td><td>[x; y; z; vx; vy; vz]</td></tr><tr><td>false</td><td>[x; y; z]</td></tr></table></td></tr>
<tr><td>'Sensor spherical'</td><td>Coordinate Dependence on HasRangeRate and HasElevation<table><tr><th>HasRangeRate</th><th>HasElevation</th><th>Coordinates</th></tr><tr><td>true</td><td>true</td><td>[az; el; rng; rr]</td></tr><tr><td>true</td><td>false</td><td>[az; rng; rr]</td></tr><tr><td>false</td><td>true</td><td>[az; el; rng]</td></tr><tr><td>false</td><td>false</td><td>[az; rng]</td></tr></table></td></tr>
</table>

Measurement Parameters

The `MeasurementParameters` field consists of an array of structures that describe a sequence of coordinate transformations from a child frame to a parent frame or the inverse transformations (see “Frame Rotation”). The longest possible sequence of transformations is Sensor → Platform → Scenario. For example, if the detections are reported in sensor spherical coordinates and `HasINS` is set to `false`, then the sequence consists of one transformation from sensor to platform. If `HasINS` is `true`, the sequence of transformations consists of two transformations – first to platform coordinates then to scenario coordinates. Trivially, if the detections are reported in platform rectangular coordinates and `HasINS` is set to `false`, the transformation consists only of the identity.

The structure fields are shown here. Not all fields have to be present in the structure. The set of fields and their default values can depend on the type of sensor.

Field	Description

`Frame`	Enumerated type indicating the frame used to report measurements. When detections are reported using a rectangular coordinate system, `Frame` is set to `'rectangular'`. When detections are reported in spherical coordinates, `Frame` is set `'spherical'` for the first `struct`.
`OriginPosition`	Position offset of the origin of the frame relative to the parent frame, represented as a 3-by-1 vector.
`OriginVelocity`	Velocity offset of the origin of the frame relative to the parent frame, represented as a 3-by-1 vector.
`Orientation`	3-by-3 real-valued orthonormal frame rotation matrix.
`IsParentToChild`	A logical scalar indicating if `Orientation` performs a frame rotation from the parent coordinate frame to the child coordinate frame. If `false`, `Orientation` performs a frame rotation from the child coordinate frame to the parent coordinate frame.
`HasElevation`	A logical scalar indicating if elevation is included in the measurement. For measurements reported in a rectangular frame, and if `HasElevation` is `false`, the measurements are reported assuming 0 degrees of elevation.
`HasAzimuth`	A logical scalar indication if azimuth is included in the measurement.
`HasRange`	A logical scalar indication if range is included in the measurement.

`HasVelocity`	A logical scalar indicating if the reported detections include velocity measurements. For measurements reported in the rectangular frame, if `HasVelocity` is `false`, the measurements are reported as `[x y z]`. If `HasVelocity` is `true`, measurements are reported as `[x y z vx vy vz]`.

Object Attributes

Object attributes contain additional information about a detection:

Attribute	**Description**
`TargetIndex`	Identifier of the platform, `PlatformID`, that generated the detection. For false alarms, this value is negative.
`SNR`	Detection signal-to-noise ratio in dB.

Signal Structure

Emitted signals have this structure:

Field	Description
`PlatformID`	`1`
`EmitterIndex`	`1`
`OriginPosition`	real-valued 3-by-1 vector
`OriginVelocity`	real-valued 3-by-1 vector
`Orientation`	1-by-1 quaternion
`FieldOfView`	`[1 5]`
`EIRP`	`100`
`RCS`	`0`
`CenterFrequency`	`300e6`
`Bandwidth`	`3e6`
`WaveformType`	`0`
`ProcessingGain`	`0`
`PropagationRange`	`0`
`PropagationRangeRate`	`0`
`IsDirectPath`	`1`

INS

INS is valid when the `HasINS` property is `true`.

Platform pose information from an inertial navigation system (INS) is a structure which has these fields:

Field	Definition
`Position`	Position of the GPS receiver in the local NED coordinate system specified as a real-valued 1-by-3 vector. Units are in meters.
`Velocity`	Velocity of the GPS receiver in the local NED coordinate system specified as a real-valued 1-by-3 vector. Units are in meters per second.
`Orientation`	Orientation of the INS with respect to the local NED coordinate system specified as a scalar quaternion or a 3-by-3 real-valued orthonormal frame rotation matrix. Defines the frame rotation from the local NED coordinate system to the current INS body coordinate system. This is also referred to as a "parent to child" rotation.

Sensor Configuration

Field	Description
`SensorIndex`	Unique sensor index
`IsValidTime`	Valid detection time, returned as `0` or `1`. `IsValidTime` is `0` when detection updates are requested at times that are between update intervals specified by `UpdateInterval`.
`IsScanDone`	`IsScanDone` is `true` when the sensor has completed a scan.
`FieldOfView`	Field of view of sensor determines which objects fall within the sensor beam during object execution. The field of view is defined as a 2-by-1 vector of positive real values, [azfov;elfov].
`MeasurementParameters`	`MeasurementParameters` is an array of structures containing the coordinate frame transforms needed to transform positions and velocities in the top-level frame to the current sensor frame.

Emitter Configuration

Field	Description
`EmitterIndex`	Unique emitter index
`IsValidTime`	Valid emission time, returned as `0` or `1`. `IsValidTime` is `0` when emitter updates are requested at times that are between update intervals specified by `UpdateInterval`.
`IsScanDone`	`IsScanDone` is `true` when the emitter has completed a scan.
`FieldOfView`	Field of view of emitter.
`MeasurementParameters`	`MeasurementParameters` is an array of structures containing the coordinate frame transforms needed to transform positions and velocities in the top-level frame to the current emitter frame.